

Instructions: Complete each of the following exercises for practice.

1. Compute the curl and divergence of the vector field $\mathbf{F}$.

(a) $\mathbf{F} = \langle xy^2z^2, x^2yz^2, x^2y^2z \rangle$

(b) $\mathbf{F} = \langle 0, x^3yz^2, y^4z^3 \rangle$

(c) $\mathbf{F} = \langle xye^z, 0, yze^x \rangle$

(d) $\mathbf{F} = \langle \sin(yz), \sin(zx), \sin(xy) \rangle$

(e) $\mathbf{F} = \left\langle \frac{\sqrt{x}}{1+z}, \frac{\sqrt{y}}{1+x}, \frac{\sqrt{z}}{1+y} \right\rangle$

(f) $\mathbf{F} = \langle \ln(2y+3z), \ln(x+3z), \ln(x+2y) \rangle$

(g) $\mathbf{F} = \langle e^x \sin(y), e^y \sin(z), e^z \sin(x) \rangle$

(h) $\mathbf{F} = \langle \arctan(xy), \arctan(yz), \arctan(zx) \rangle$

2. Determine whether or not $\mathbf{F}$ is conservative. If so, compute a potential f .

(a) $\mathbf{F} = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

(b) $\mathbf{F} = \langle xyz^4, x^2z^4, 4x^2yz^3 \rangle$

(c) $\mathbf{F} = \langle z \cos(y), xz \sin(y), x \cos(y) \rangle$

(d) $\mathbf{F} = \langle 1, \sin(z), y \cos(z) \rangle$

(e) $\mathbf{F} = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$

(f) $\mathbf{F} = \langle e^x \sin(yz), ze^x \cos(yz), ye^x \cos(yz) \rangle$

3. Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ for which $\text{curl}(\mathbf{G}) = \mathbf{F}$? Why?

(a) $\mathbf{F} = \langle x \sin(y), \cos(y), z - xy \rangle$

(b) $\mathbf{F} = \langle x, y, z \rangle$

4. Prove every vector field of form $\mathbf{F}(x, y, z) = \langle f(x), g(y), h(z) \rangle$ is *irrotational* (i.e. $\text{curl}(\mathbf{F}) = \mathbf{0}$).

5. Prove every vector field of form $\mathbf{F}(x, y, z) = \langle f(y, z), g(x, z), h(x, y) \rangle$ is *incompressible* (i.e. $\text{div}(\mathbf{F}) = 0$).

6. Prove each of the following identities for scalar field $\alpha(x, y, z)$ vector fields $\mathbf{F}(x, y, z)$ and $\mathbf{G}(x, y, z)$.

(a) $\text{div}(\mathbf{F} + \mathbf{G}) = \text{div}(\mathbf{F}) + \text{div}(\mathbf{G})$

(b) $\text{curl}(\mathbf{F} + \mathbf{G}) = \text{curl}(\mathbf{F}) + \text{curl}(\mathbf{G})$

(c) $\text{div}(\alpha \mathbf{F}) = \alpha \text{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla \alpha$

(d) $\text{curl}(\alpha \mathbf{F}) = \alpha \text{curl}(\mathbf{F}) + (\nabla \alpha) \times \mathbf{F}$

(e) $\text{div}(\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \text{curl}(\mathbf{F}) - \mathbf{F} \cdot \text{curl}(\mathbf{G})$

(f) $\text{curl}(\text{curl}(\mathbf{F})) = \nabla(\text{div} \mathbf{F}) - \nabla^2 \mathbf{F}$

7. Prove every continuous function $f(x, y, z)$ is the divergence of some vector field.